Journal of the European Ceramic Society **19** (1999) 2957–2962

PII: S0955-2219(99)00087-4
0955-2219/99/$ - see front matter

Weibull Statistical Analysis of the Mechanical Strength of a Glass Eroded by Sand Blasting

M. A. Madjoubi,* C. Bousbaa, M. Hamidouche and N. Bouaouadja

Laboratory of Materials, IOMP, Ferhat Abbas University, Sétif 19000, Algeria

(Received 28 January 1999; accepted 14 March 1999)

Abstract

In Saharian regions, the erosion of glass by sand particles during sandstorms is a regular phenomenon. The progressive loss of matter on surface affects both the optical transmission and mechanical strength. In this work, the influence of sand impacts on glass strength was simulated in laboratory. We used Weibull distribution function to characterize statistically the variation of the mechanical strength of a soda-lime glass in the as received state and eroded by sand blasting during 30 and 60 min. From the failure probabilities distributions, we notice an important drop in strength values (about 13%) after 30 min and a tendency to level out with a much reduced dispersion after 60 min. The Weibull plots for the as-received state and for the 30 min eroded state present curves with a knee. They were considered as bimodal forms (two straigth lines) denoting the presence of two kinds of defects that control strength. The Weibull plot for the 60 mins eroded state sample presents one straight line (unimodal form) that indicates the predominance of erosion defects. From micrographical observations on eroded specimen, we observed a tendency toward a damaging homogeneity of the surfaces exposed to sand blasting. This explains the uniformity of the strength values obtained after 1 h of sand blasting.

Keywords: glass, mechanical properties, strength, erosion, sand blasting.

1 Introduction

Under the same conditions of specimen preparation and loading, the experimental evaluation of glass mechanical strength gives very scattered values (see Fig. 1) that necessitate a statistical analysis. The coefficient of variation about the mean can reach more than 20%. The strength values dispersion is inherently tied to the distribution of surface flaws introduced during glass processing, by mechanical contact during the use of glass or induced by agressive environments such as sand blasting, chemical attacks, etc.

Among statistical distribution functions, the Weibull model is the most widely used to analyse statistically strength measurements and life time predictions of glass components. It is very suitable by its simple mathematical form and by its adaptability to experimental data. Since its publication in 1939, it has been applied by numbrous research workers on different brittle materials (ceramics) to characterize strength and lifetime variability under various loading conditions.[1–5]

The tests made on brittle materials revealed that their fracture strength depends essentially on the existence of volumetric or surface flaws that acts as stress intensifiers. The fracture occurs when the stress at any flaw is sufficient to cause unstable crack propagation. On a uniformaly stressed specimen, it is the most critical flaw (by its size, form and position) that controls strength. The probability distribution of strength corresponds then to the distribution of critical flaws. A theoretical analysis made by Jayatilaka and Trustrum[6] has shown the relation between different possible flaw size distribution following an inverse power law to the Weibull distribution. The Weibull's model, based on 'the weakest link concept' considers the material structure as a chain whose strength is controlled by its weakest link which is equivalent to the region with the largest flaw.[2]

According to this theory as presented by Varashneya,[7] the cumulative probability of failure P of a body is given by:

$$P = 1 - \mathrm{Exp}[-R] \qquad (1)$$

* To whom correspondence should be addressed.

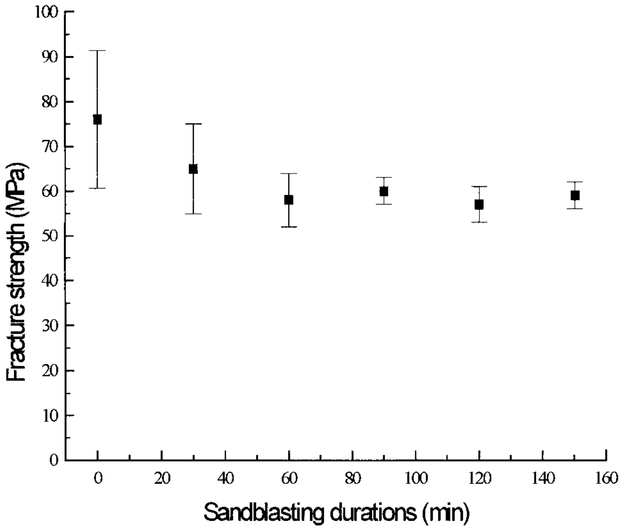

Fig. 1. Variation of the fracture strength versus sand blasting durations.

where R is the risk of rupture defined for a volume V as

$$R = \int_{V} [(\sigma - \sigma_U)/\sigma_0]^{m} dV \quad \text{for} \quad \sigma > \sigma_U \qquad (2)$$

$$R = 0 \qquad \text{for}\, \sigma < \sigma_U \qquad (3)$$

σ is the stress applied on the element dV; σ_U is the threshold stress, i.e. the minimum stress that can cause failure; σ_0 is a scaling or normalizing parameter whose unit is the unit of: stress×volume$^{1/m}$, and m is the Weibull modulus, also called the shape parameter.

The Weibull modulus characterizes the strength distribution. As this number gets larger, the distribution narrows. Typical values of m for soda-lime glass are between 5 and 15. The threshold stress σ_U is usually set equal to zero to obtain, after integrating R, a two parameter function [Eq. (7)] easier to linearize without compromising the results. It was shown indeed by Trustrum and Jayatilaka[8] that setting σ_U equal to zero gives more conservative results (overestimation of failure probabilities) without any major change in the overall cumulative failure probability distribution.

R can be written in a simple integrated form as:

$$R = Y_V.V(\sigma/\sigma_0)^{m} \qquad (4)$$

where Y_V is a factor that depends on the loading conditions. It is equal to 1 in tensile loading where V is uniformly stressed.

The product $Y_V.V$ represents the effective volume, i.e. the volume of an equivalent specimen tested in tension that would have the same probability of fracture as that tested in another loading conditions (3- or 4-point bending).

For glasses whose strengths are mainly controlled by surface flaws, the expression of R becomes:

$$R = Y_S.S(\sigma/\sigma_0)^{m} \qquad (5)$$

where $Y_S.S$ is the effective surface.

For 4-point bending test, Y_S has been defined by integration as:

$$Y_S = [m(L_i/L_0) + 1] \qquad (6)$$

$$\left[(w/h) + \left\{1/(m+1)\right\}\right]/2\left[\left\{1 + (w/h)\right\}\{m+1\}\right]$$

where L_i is the inner span, L_0 the outer span, w the width and h the height of the specimen.

The loading factor Y_S and the surface S are taken as part of the σ_0 parameter redefined with the unit of stress (MPa) such that the Weibull distribution function becomes:

$$P = 1 - \text{Exp}[-(\sigma/\sigma_0)^{m}] \qquad (7)$$

By taking twice the logarithms of the survival probability $(1 - P)$, we obtain the equation:

$$\text{LnLn}[1/(1-P)] = m.\text{Ln}(\sigma) + \text{Ln}[(1/\sigma_0)^{m}] \qquad (8)$$

This equation can be plotted as a straight line LnLn$[1/(1-P)]$ versus Ln(σ) whose slope is the Weibull modulus m and whose intercept at the origin is Ln$[(1/\sigma_0)^{m}]$.

The parameters m and σ_0 defining the Weibull distribution [eqn. (7)] are usually determined either graphically or numerically by the least squares method. For that purpose, we must assign a failure probability P_i to each value of σ_i after ranking all the measured values in ascending order ($_i$ taking value from 1 to n which corresponds to the number of measurements of the sample tested) according to one of the principal probability estimators used:

$$P_i = i/(n+1) \qquad (9)$$

$$P_i = (i - 0{\cdot}5)/n \qquad (10)$$

$$P_i = (i - 0{\cdot}3)/(n + 0{\cdot}4) \qquad (11)$$

Although it was shown by numerical simulation that the second [eqn. (10)] and the third [eqn. (11)] estimators give more precise results, the first estimator [eqn. (9)] is still frequently used for design purposes as it gives more conservative estimates of Weibull modulus.[9]

The parameters m and σ_0 can also be determined with more precision using non-linear equation methods[10] based on fitting the data σ_i to the non-linear [eqn. (7)]. Among those methods, we have the method of moments, the maximum likelihood method and the direct non-linear least squares analysis. From different studies made by simulation (using Monte Carlo method) on the efficiency of these different methods, it appears that the maximum likelihood method gives the least biased results[8,11] on the parameter m.

Among the assumptions considered in the use of Weibull model is that the material is an homogeneous medium with one population of flaws randomly distributed in sufficient number within all the specimens. In the case of the presence of two different flaw populations that control strength, the Weibull plot [$\mathrm{LnLn}[1/(1-P)$ versus $\mathrm{Ln}(\sigma)$] will have a bimodal form (i.e. two straight lines plot). To clearly characterize the presence of a second flaw population, it is recommended to use several samples with different sized tests specimens.[12] This would give for one flaw population a uniform parallel shift of the strength distribution on a conventional failure cumulative probability graph according to the Weibull size scaling relationship:

$$\sigma_2/\sigma_1 = (S_1/S_2)^{1/\mathrm{m}} \tag{12}$$

This relation says that a specimen of effective surface S_2 would have the same failure probability as a specimen of surface S_1 if the stress is changed from σ_1 to σ_2. If there is a second flaw population, the strength distribution would shift in a non-parallel and non-uniform way with a change in specimen size. When it is possible, the use of fractography for visualizing fracture origins can also help to identify the flaw population that control strength.

In the case of bimodal form where failure origins are known by fractography, methods such as the censored data technique[13] or the maximum likelihood technique[14] could be used for characterizing the Weibull parameters for the concurred flaws populations (e.g. edge and surface flaws in glass).

2 Experimental Procedures

2.1 Materials

The glass used is a silica soda lime glass[15] manufactured by Fourcault drawing process in E.N.A.V.A. (an Algerian company located at Jijel region). Its mean chemical composition and some of its physical properties are given in Tables 1 and 2.

Table 1. Mean chemical composition of the glass used[15]

Composition	SiO_2	CaO	MgO	Na_2O	Al_2O_3	*Others*
% weight	72·2	6·7	4·0	15·0	1·9	0·2

Table 2. Some physical properties of the glass used[15]

Properties	*Values*
Elastic modulus	72 GPa
Poisson's ratio	0·22
Mechanical strength	78 MPa
Hardness	4·5 GPa
Density	2·52 g cm^{-3}
Transition temperature	530°C
Optical transmission	91·5%

2.2 Equipment used and procedure

The glass erosion simulation tests by sand blasting were undertaken using a sand blower apparatus. The erosion tests were carried out with a stationary target impacted by sand particles accelerated in an air stream by a ventilator. The air blower velocity was measured using an anemometer and was found to be 16·60 m s^{-1}. The sand feed during the erosion tests was about 1·66 g s^{-1} and the impingement angle was fixed constant at 90° (specimens surface are perpendicular to the air flow). The distance between the pipe convergent nozzle and the specimens was adjusted to 25 cm in such a way that the particle velocity becomes nearly constant at the approach of the target. During the tests, the sand used was washed and dried in order to eliminate the dust (mean particles size $< 100\ \mu$m).

The specimens, in form of small slabs of dimensions 100×10×3 mm^3, were visually examined in order to eliminate those containing eventual apparent prior contact flaws and chamfered on the tension sxurface to reduce edge flaws effect during strength tests. Three samples of 50 specimens each were systematically tested on a four-point bending rig. One sample was tested as a reference keeping the specimens in their as-received state unexposed to sand blasting. The second and the third samples were respectively eroded by sandblasting in the same conditions during 30 and 60 min before the bending tests. Some micrographical observations of the typical fractured specimens and the eroded surfaces were made using a Neophot microscope.

The *seeked* objective from these tests in the statistical analysis is to compare the strength probability distribution and the Weibull plots for the three samples. But before that, in order to observe the general evolution of the fracture strength with the sand blasting durations, some preliminary tests were made on small samples ($n = 6$) for different times up to 150 min (see Fig. 1).

3 Results and Discussion

The bending results of the preliminary tests are presented in Fig. 1. Two principal remarks can be made from these results. We, first of all, notice a sharp drop in strength within an hour duration followed by almost constant level of strength values. The second remark concerns the decrease of the standard deviation from 0 min duration (unexposed specimens to erosion tests). There is less values scattering with time.

After achieving the bending tests on the three samples for the as-received state and for the 30 and 60 min sand blasting durations, we ordered the strength values for each sample and assigned a probability P_i to each value σ_i using the estimator $P_i = 1/(n+1)$. The different cumulative probabilities distribution are shown in Fig. 2. For a 50% failure probability, the strength decreases from 76 MPa for the as-received state to 66 MPa after 30 min and to 64 MPa after 60 min durations. With the exception of a few high extreme values, the distributions for the eroded glass during 30 and 60 min are comparably closer to each other.

In Fig. 3, we have represented the three different Weibull plots. We can observe that the bimodal character of the lines is clearly apparent for the initial state ($t = 0$ min) and tends to disappear after 30 and then 60 min of sand blasting. The Weibull plot for the as-received state ($t = 0$ min) shows two straight line sections. This bimodal distribution indicates the presence of two flaws families that are responsible for the strength distribution. There is a lower line whose slope gives a Weibull modulus $m = 24{\cdot}2$ concerning the weakeast strength values, caused probably by predominant defects introduced during specimen preparation. These could be severe edge flaws remaining after cutting and chamfering operations. The upper line with Weibull modulus $m = 4{\cdot}2$ covers a large dispersion of strength values corresponding to the distribution of surface flaws left after the processing or induced by mechanical contact. For the damaged surface during 30 min, the Weibull plot presents also a bimodal form with a lower line ($m = 18{\cdot}2$) and an upper line ($m = 6{\cdot}3$). Here also, we have two statistical flaws families that intervene in the strength distribution. The lower values part could be caused by erosion exclusively as all these values seem to be weaker than the lowest extreme value obtained for the as-received state. The higher values part with a Weibull modulus comparable to that of the as-received glass would then be caused by other surface flaws. The almost straight line for the 60 min duration with a high modulus ($m = 10{\cdot}46$) indicates the predominance of erosion defects and a narrow values dispersion.

The microscopic observations [Fig. 4(a) and 4(b)] present the surface damage caused by erosion respectively for 30 and 60 min durations. The micrographs show the formation of individual defects that group and then extend progressively on all the surface when the duration increases. We can clearly observe that there is a tendency to an homogeneization of the surface damaging, which explains the reduced values dispersion seen with sand blasting durations in Fig. 1.

Figure 5 shows some details of typical surface damage induced by sand impacts. The damage is

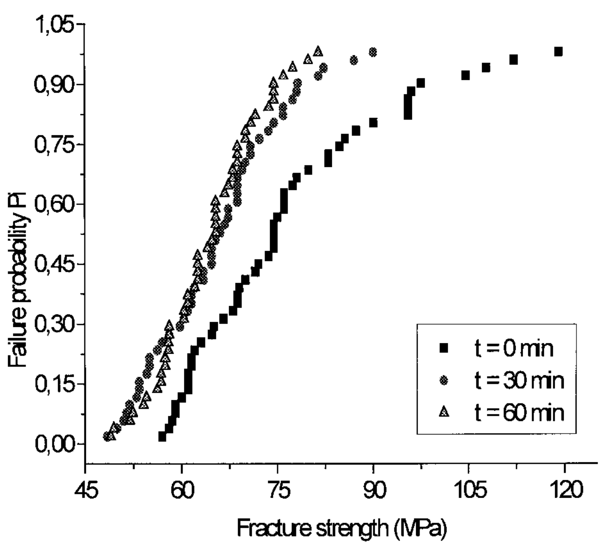

Fig. 2. Variation of the failure probability versus the fracture strength.

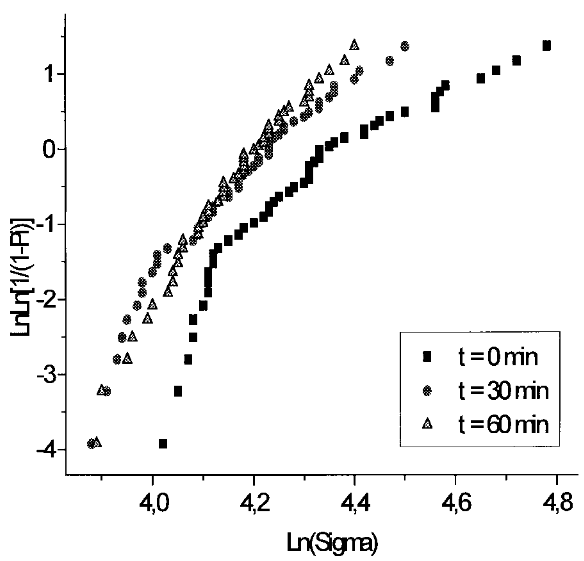

Fig. 3. Weibull plots established for three different states: as-received state ($t = 0$ min) and sand blasted for 30 and 60 min.

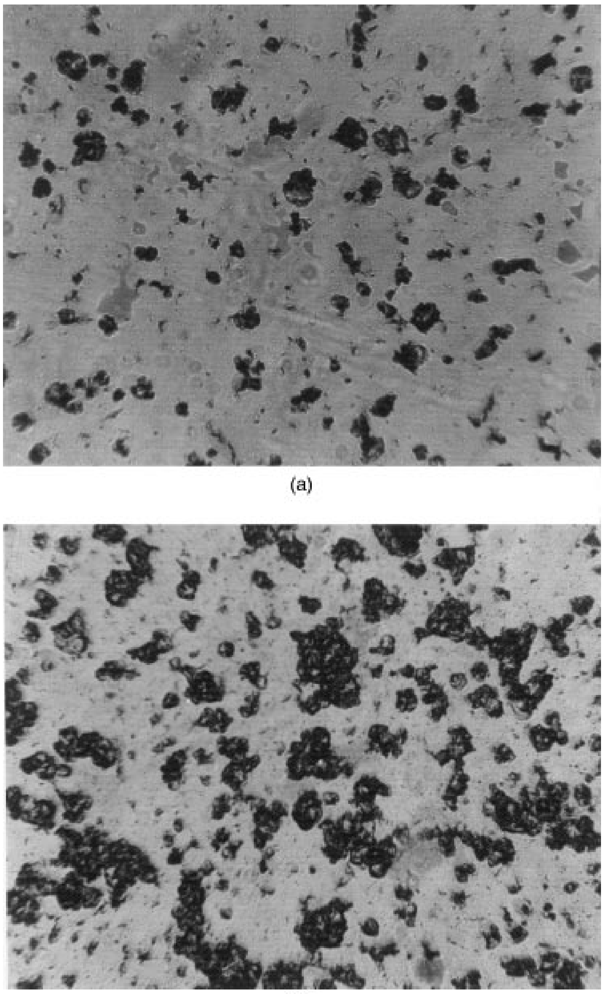

Fig. 4. Micrographs of the eroded surfaces showing the damage caused by the sand particle impacts after (a) 30 min, (b) 60 min (×82).

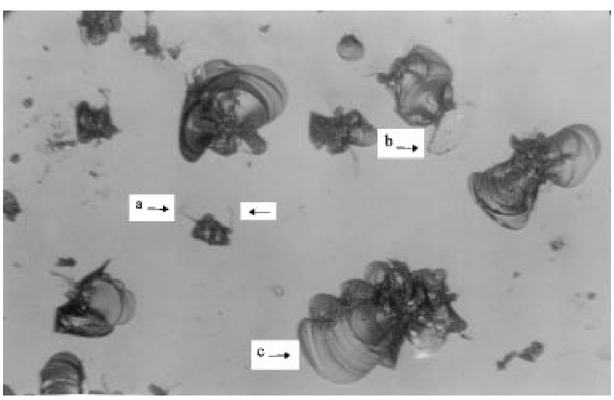

Fig. 5. Micrograph of the damaged surfaces showing some details (arrows a, b, c) of the formation sequences of lateral cracks (×320).

essentially produced by scaling with formation and extension of lateral cracks corresponding to sharp indentation damage. We can show, for example, the trace of lateral cracks which are nearly parallel to the surface (arrow a), the cracks which curve up and intersect the glass surface (arrow b) and finally the morphology of the scales after detachment (arrow c). Conical fractures typical of Hertzian indentation (blunt indentation damage) were also noticed.[16]

4 Conclusion

A statistical analysis based on Weibull's model helped us to follow and to describe the variation of the mechanical strength of a soda-lime glass eroded by sand blasting during 30 and 60 min. A sharp decrease in strength happens during the first 30 min. The corresponding Weibull plot show a bimodal form denoting the distribution between the severe defects caused by erosion and those less severe left after the specimen preparation. After 1 h duration of sand blasting, a straight line is obtained with a high Weibull modulus ($m = 10{\cdot}46$) characterizing the predominance of erosion defects that give a narrow dispersion of fracture strength values. The reduced strength scatter shows that the cumulate sand blasting erosion induced a distribution of severe flaws which is approaching uniformity in comparison with that of the as-received state flaws. The tendency towards an homogeneous damaged surface has been revealed from microscopic observations. Besides, the closeness of the two strength distributions for 30 and 60 min durations suggests that there is no further strength degradation after 30 min.

References

1. Evans, A. G. and Langdon, T. G., Structural ceramics. *Prog. Mater. Sci.*, Vol. 21. Pergamon Press, Elmsford, NY, 1976, pp. 171–425.
2. Davidge, R. W., *Mechanical Behaviour of Ceramics*. Cambridge University Press, Cambridge, 1979, (chapter 9).
3. Wiederhorn, S. M. and Fuller, E. R., Structural reliability of ceramic materials. *Mater. Sci. Eng.*, 1985, **71**, 169.
4. Petrovic, J. J. and Stout, M. G., Fracture of Al_2O_3 in combined tension/torsion: II, Weibull theory. *J. Am. Ceram. Soc.*, 1981, **64**, 661–666.
5. Williams, R. M. and Swank, L. R., Use of Weibull statistics to correlate MOR ball on ring and rotational fast fracture tests. *J. Am. Ceram. Soc.*, 1983, **66**, 765–768.
6. Jayatilaka, A. S. and Trustrum, K., Statistical approach to fracture. *J. Mater. Sci.*, 1977, **12**, 1426–1430.
7. Varashneya, A. R., Fundamentals of inorganic glasses. New York State College of Ceramics, Alfred University, New York, 1994.
8. Trustrum, K. and Jayatilaka, A. S., On estimating the Weibull modulus for a brittle materials. *J. Mater. Sci*, 1979, **14**, 1080–1084.
9. Bergman, B., On the estimation of the Weibull modulus. *J. Mater. Sci. Letters*, 1984, **3**, 689–692.
10. Steen, M., Sinnema, S. and Bressers, J., Statistical analysis of bend strength data according to different evaluation methods. *J. Eur. Ceram. Soc.*, 1992, **9**, 437–445.
11. Langlois, R., Estimation of Weibull parameters. *J. Mater. Sci. Letters*, 1991, **10**, 1049–1051.

12. Quinn, G. D., Design data for engeneering ceramics: a review of the flexure tests. *J. Am. Ceram. Soc.*, 1991, **74**, 2037–2066.
13. Jakus, K., Ritter, J. E., Service, T. and Sonderman, D., Evaluation of bimodal concurrent flaw distributions. *J. Am. Ceram. Soc.*, 1981, **64**, 174–175.
14. Sonderman, D., Jakus, K., Ritter, J. E., Yuhaski, S. and Service, T. H., Maximum likelihood estimation techniques for concurrent flaw subpopulations. *J. Mater. Sci.*, 1985, **20**, 20–23.
15. Bousbaa, C., Iferroudjene, N., Bouzid, S. and Bouaouadja, N., Effects of durations of sand blasting on the properties of window glass. *Glass Techn.*, 1998, **39**, 24–26.
16. Lawn, B. R., Fracture of brittle solids, 2nd Edn. Cambridge University Press, Cambridge (chapter 8).